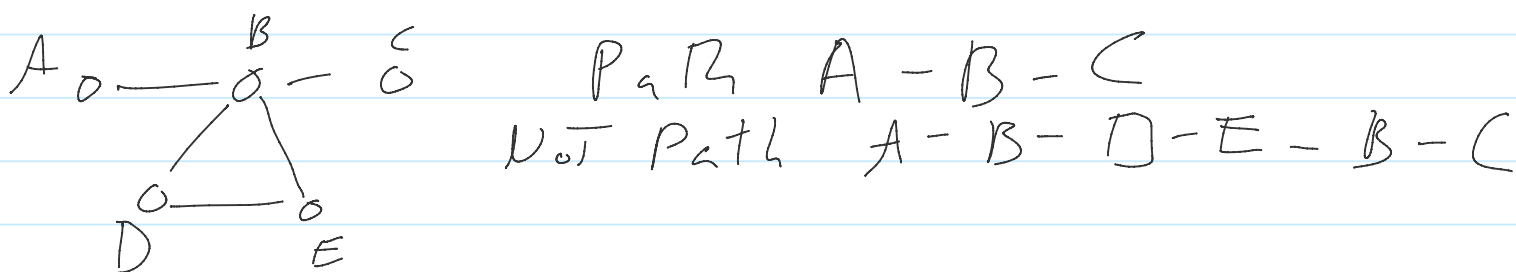


# 1/29 Graph def and Prim's Proof

Monday, January 29, 2018 6:16 PM

Path is a sequence of Edges which connects a sequence of Vertices such that the Vertices are distinct



A graph is connected iff  $\forall a, b \in V(G)$   
 $\exists$  a path from a to b.

A tree is a connected graph which contains no cycles

A cycle is a sequence of edges that connect a sequence of vertices such that only the first and last vertex is reused and the first vertex is the last vertex

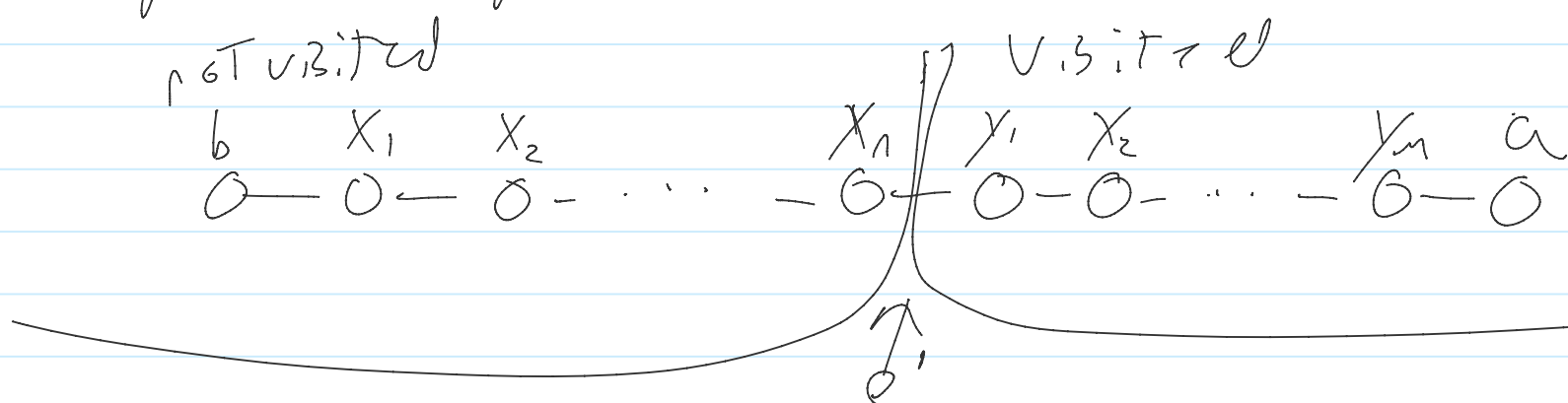
Suppose for the sake of contradiction that there exist a Spanning Tree,  $T'$  that has a better edge sum than our resulting Spanning Tree  $T$ .

Let  $e$  be the first edge built using Prim's alg. that is not in  $T'$

Let  $a, b$  be the endpoints of  $e$ .

WLOG Let  $a$  be in our Visited set and  $b$  be not in our Visited set

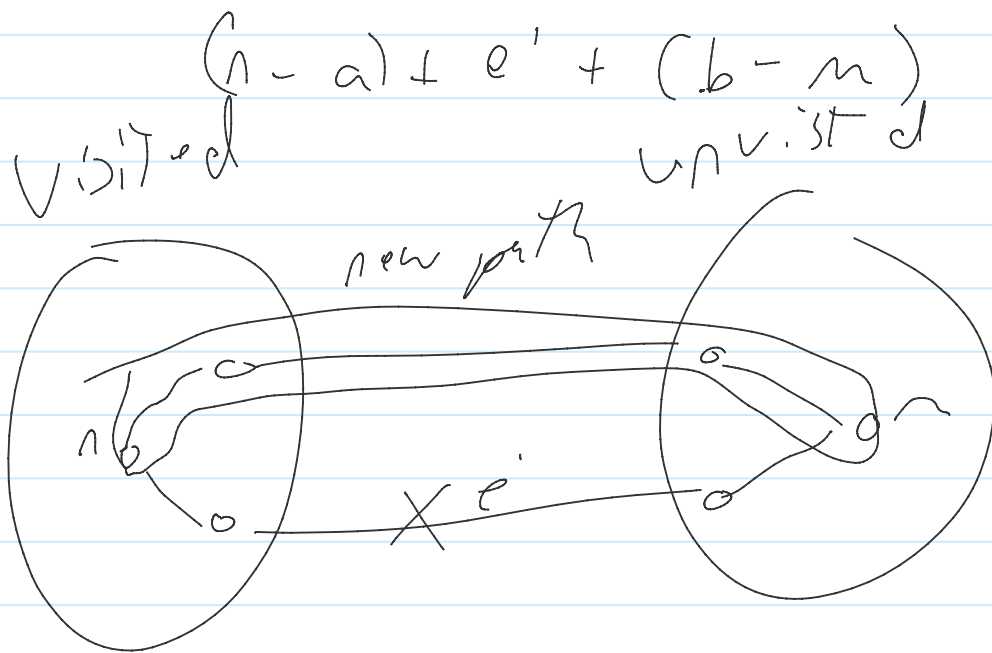
Let  $p$  be the path from  $b$  to  $a$  in  $T'$



This edge has a weight greater than the weight of  $e$

$$T'' = T' - e' + e$$

if we want to construct path from  $n$  to  $m$   
 and path in  $T'$  crosses  $e'$  then  
 WLG  $(n-a)$   $(m-b)$



$$\text{weight sum of } T'' = \text{weight sum } T' + e \text{ weight} - e' \text{ weight}$$

$$< \text{weight sum } T'$$

